Broadband efficient nonlinear difference generation in a counterpropagating configuration

Jordi Martorell[a)]
Institut de Ciències Fotòniques, C/ Jordi Girona, 29, 08034 Barcelona, Spain and Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya, C/Colom, 11, 08222 Terrassa (Barcelona), Spain

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A counterpropagating parametric nonlinear interaction is considered in the framework of a short one-dimensional photonic crystal. This interaction, otherwise highly inefficient, is shown to be efficient in the framework of such one-dimensional photonic crystal. The large momentum mismatch of this counterpropagating interaction is shown to be compensated for a broad range of frequencies when difference generation is considered. The numerical results presented indicate that such nonlinear photonic crystals are a very good material to consider the observation of backward parametric oscillation without mirror feedback. © *2005 American Institute of Physics*.
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The second order nonlinear interaction began to be incorporated in the field of photonic crystals (PCs) when second harmonic generation in three-dimensional (3D) PCs was observed in 1995.[1] In that early work and subsequent publications it was shown that the periodic distribution of dielectric material in PCs was very useful to phase match or enhance such nonlinear interaction.[1–10] Among the effects reported to date, a unique feature of PC lattices stands out, which is the ability to reach a second order nonlinear interaction even when the macroscopic crystalline structure is centrosymmetric.[1,4,5,9]

Another very unique feature of PC structures is the possibility to hold an efficient nondegenerate parametric interaction of counterpropagating waves. In the bulk of the majority of conventional nonlinear materials used for parametric generation, such counterpropagating interaction is extremely out of phase matching and, consequently, transfer of energy between the three fields involved vanishes. In fact, the fundamental nonlinear process of backward parametric oscillation without mirror feedback,[11,12] requires a phase matching mechanism capable of compensating such large mismatch.[13] Certain nonlinear PCs are built as periodic arrays of very short nonlinear crystals, separated by a linear material. In such a configuration, there are a large number of interfaces where the counterpropagating interaction may be as significant as the copropagating one.[14] I will show that, in certain conditions, such counterpropagating interaction is perfectly phase matched and leads to an efficient transfer of energy among the three fields at the three different frequencies.

I consider a one-dimensional (1D) PC made of dielectric layers of thickness b separated with a homogeneous material of a lower dielectric constant and thickness a, contained in the xy plane. The nonlinear material is assumed to be homogeneously spread within the layer of thickness b. Similar 1D PCs have, in fact, already been fabricated and tested for copropagating second harmonic generation.[9] As shown in Fig. 1, the geometry is such that the wave vector for the three fields is parallel to the z axis while the electric field vector is on the xy plane. It is assumed that there are two strong beams, one incident from the left and another incident from the right, at frequencies ω_3 and ω_2, respectively. A third beam is generated either to the left or to the right (cf. Fig. 1) at $\omega_1=\omega_3-\omega_2$ when we consider difference generation. Because of multiple reflections within the PC, at each given nonlinear layer of the set (except for the first and last), there is also a field at ω_3 incident from the right and a field at ω_2 incident from the left (cf. Fig. 1). Unless we consider a very long PC one may neglect the depletion of the two pump beams. One should write, then, an equation for the two counterpropagating beams at ω_1. Under the assumption of harmonic solutions, the electric field amplitude wave equation at ω_1 becomes

$$\nabla^2\mathbf{E}_{\omega_1}+\frac{\omega_1^2}{c^2}\frac{\varepsilon(\mathbf{r})}{\varepsilon_o}\mathbf{E}_{\omega_1}=-\mu_o\omega_1^2\mathbf{P}^{nl}_{\omega_1}-\frac{1}{\varepsilon(\mathbf{r})}\nabla(\nabla\cdot\mathbf{P}^{nl}_{\omega_1}), \tag{1}$$

where $\varepsilon(\mathbf{r})$ is the dielectric constant a $\mathbf{P}^{nl}$ the nonlinear polarization source. For difference generation, the nonlinear polarization source term may be separated into eight different contributions that can be paired into four equations:

$$\mathbf{P}^{nl\pm}_{\omega_1}(\mathbf{r})=\varepsilon_o\chi^{(2)}(\mathbf{r}):\mathbf{E}^{+}_{\omega_3}(\mathbf{r})\mathbf{E}^{-*}_{\omega_2}(\mathbf{r}), \tag{2a}$$

$$\mathbf{P}^{nl\pm}_{\omega_1}(\mathbf{r})=\varepsilon_o\chi^{(2)}(\mathbf{r}):\mathbf{E}^{+}_{\omega_3}(\mathbf{r})\mathbf{E}^{+*}_{\omega_2}(\mathbf{r}), \tag{2b}$$

$$\mathbf{P}^{nl\pm}_{\omega_1}(\mathbf{r})=\varepsilon_o\chi^{(2)}(\mathbf{r}):\mathbf{E}^{-}_{\omega_3}(\mathbf{r})\mathbf{E}^{-*}_{\omega_2}(\mathbf{r}), \tag{2c}$$

$$\mathbf{P}^{nl\pm}_{\omega_1}(\mathbf{r})=\varepsilon_o\chi^{(2)}(\mathbf{r}):\mathbf{E}^{-}_{\omega_3}(\mathbf{r})\mathbf{E}^{+*}_{\omega_2}(\mathbf{r}), \tag{2d}$$

where $\chi^{(2)}(\mathbf{r})$ is the periodic second order nonlinear susceptibility, the E_{ω_1} are the electric field components for a wave at frequency ω_1, and the "+" and "−" signs on the field amplitudes indicate z-positive and z-negative propagation directions, respectively. The "+" sign on the polarization indicates contribution to the field at ω_1 propagating in the z-positive direction, while the "−" sign indicates contribution to the same field propagating in the z-negative direction. In commonly used bulk nonlinear crystals of a length of

[a)]Electronic mail: jordi.martorell@icfo.es

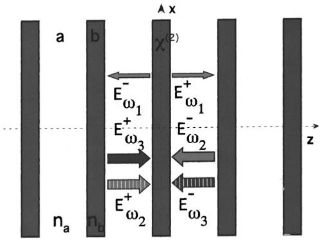

FIG. 1. Schematic representation of the z-positive and z-negative propagating fields that are involved in the nonlinear interaction we consider. The thick solid arrows represent the incident fields, while the dashed arrows represent the fields that appear because of multiple reflections in the periodic structure. The thin arrows represent the fields generated via the nonlinear interaction.

millimeters or centimeters only the source terms given in Eq. (2a) would survive and its contribution to conversion at ω_1 would be negligible.

One may solve the nonlinear wave equation for the transverse part of the electric field applying the Green function integration method developed in Refs. 15 and 16 including the four nonlinear source terms from Eqs. (2a)–(2d). Without loss of generality one may assume that the only nonvanishing terms of the second order nonlinear susceptibility tensor are those perpendicular to the z-direction. In such case one obtains very simple analytical expressions for the amplitude of the electric field at ω_1 generated by one of the nonlinear layers. Such expressions are used together with the transfer matrix formalism, applied to all three frequencies, to determine the total generated field at ω_1 in the z-positive and z-negative directions.

In a numerical application, I consider PCs made of layers of LBO[17] separated with layers of dielectric material with the index of refraction of fused silica. I consider the wavelengths of the incident fields to be $\lambda_3=355$ nm and λ_2 may be tuned in the visible and near-infrared range. As is well known, the periodicity build-in a PC can be used for phase matching of copropagating second order nonlinear interactions at certain wavelengths. Such phase matching is achieved at the upper or lower edge of a Bragg reflection band.[4,6] This mechanism of phase matching is sufficient to compensate the small dispersion of the materials of the 1D PC under study but, by no means, is able to compensate a phase mismatch of the order of $\Delta k=[k_3-(-k_2)]\mp k_1$. In fact, the mechanism needed to compensate such large effective dispersion resulting form the counterpropagating interaction comes from an interaction in a material composed of many very short pieces of nonlinear crystals and an appropriate separation between such nonlinear pieces. In Fig. 2, difference generation is shown for z positive and negative propagation directions as a function of the thickness b of the nonlinear layer while the thickness a of the linear layer is adjusted, within a range that goes from 0 to 500 nm, for maximum conversion in the case that λ_2 is fixed at 802 nm. In the numerical calculation the total number of nonlinear layers or periods considered is 100, while $\chi^{(2)}E_{\omega_{2,3}} \approx 2.410^{-4}$. The observed conversion can be compared with the case where the linear layer is assumed to have the same index of refraction as the nonlinear layer (nonphotonic crystal case). In that case the sole contribution comes from the source terms in Eq. (2a). Note that introducing an index contrast between the linear and nonlinear layers results in an increase of the conversion of a counterpropagating interaction, that can be as large as four times for generation in the z-positive direction and more than 16 times for generation in the z-negative direction. The larger conversion observed, when a periodicity on the linear part of the susceptibility is present (photonic crystal case), is a consequence of light localization, a structural index dispersion at the edge of the ω_3 band, and a contribution to conversion from all terms of Eqs. (2). Note that as seen in Fig. 2, generation in the z-positive direction can be made almost as efficient as generation in the z-negative direction if the appropriate structure is used. The apparent unbalance between generation in one direction with respect to the opposite one for a given structure is a result of a strong modulation of the nonlinear susceptibility.[18] Such an unbalance is enhanced in materials that exhibit periodic modulation of the linear susceptibility as well.

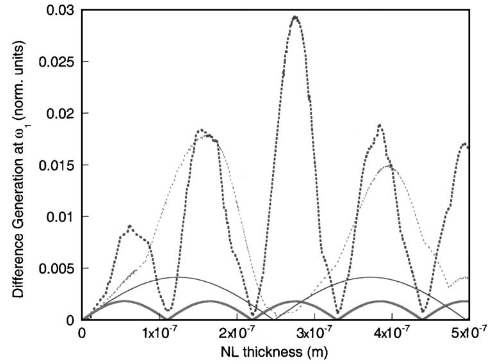

FIG. 2. Amplitude conversion for the difference frequency generation at ω_1 as a function of the thickness of the nonlinear layer in a 1D PC similar to the one schematically shown in Fig. 1, in the z-positive direction (thin dashed line), in the z-negative direction (thick dashed line), in the z-positive direction for the nonphotonic crystal case ($n_a=n_b$) (solid thin line), and in the z-negative direction also for the nonphotonic crystal case (solid thick line).

In Fig. 3, difference generation for the z-negative propagation direction is shown as a function of the wavelength λ_2, for the configuration that in Fig. 2 exhibits the maximum conversion. This configuration corresponds to a nonlinear layer thickness of 274.5 nm and a linear layer thickness of 60.02 nm. One observes that the nonlinear process can be made to be phase matched for a wide range of frequencies of

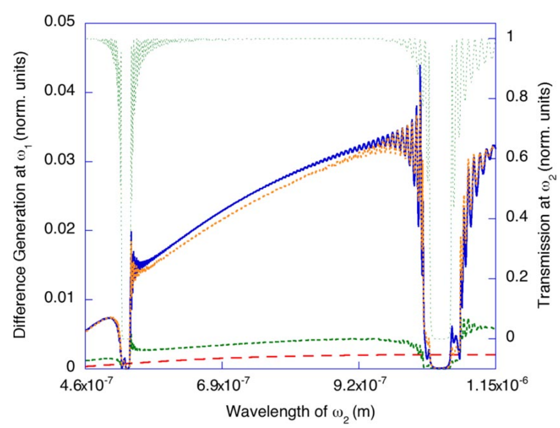

FIG. 3. Amplitude conversion for the difference frequency generation (left-hand side scale) in the z-negative direction at ω_1 as a function of the wavelength of the field at ω_2 for a 1D PC similar to the one schematically shown in Fig. 1, when all source contributions are considered (thick solid line), when only the contribution from the source term given in Eq. (2a) is nonzero (thick dotted line), when only the contribution from the source term given in Eq. (2c) is nonzero (thick dashed line), and when in the nonphotonic crystal case (thick long-dashed line). Transmission (right-hand side axis) as a function of the wavelength for the ω_2 field (thin dashed line).

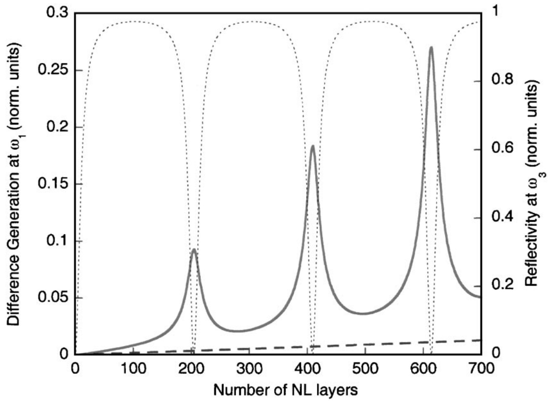

FIG. 4. Amplitude conversion for the difference frequency generation (left-hand side scale) in the z-positive direction at ω_1 as a function of the number of periods, for the same PC considered in Figs. 2 and 3, is shown at $\lambda_2 = 802$ nm (thick solid line) and when in the nonphotonic crystal (thick long-dashed line). Reflectivity (right-hand side scale) at ω_3 as a function of the number of periods (thin dashed line).

the ω_2 wave that goes from the first to the second order Bragg resonances, also shown in Fig. 3, while the frequency ω_3 is kept fixed. On the other hand, additional calculations showed that the refractive index dispersion of the materials considered has no significant effect on the efficiency of the process for the entire range of frequencies between the first and second order Bragg bands. In Fig. 3, it is also shown that when contributions from the source terms in Eqs. (2) are considered separately, the main contribution to difference generation comes from the corresponding source terms in Eq. (2a) and (2c). This last result indicates that in the photonic crystal case the nonlinear process is a combination of a cavity type enhancement with a strictly counterpropagating nonlinear interaction.

If one considers difference generation at a given wavelength as a function of the number of crystalline planes, as in Fig. 4, one observes that relative maxima and minima of the electric field amplitude exhibit an almost linear growth, indicating an almost perfect phase matching of the parametric interaction we are considering. The amplitude conversion shown in Fig. 4 exhibits some strong oscillations; Such oscillations reflect a strong field intensity within the transmission resonances at ω_3. Note that for a short number of planes, before the first resonance is reached, there is an apparent exponential growth of the conversion. Comparing the difference generation at ω_1 with the transmission at ω_3, shown also in Fig. 4, one realizes that such rapid growth in the efficiency, is an indication that a resonance is approaching as we change the number of layers in our crystal. Now, if one considers the nonphotonic crystal case, similar to the one considered in Ref. 13, by matching the indexes of the *a* and *b* layers, the counterpropagating interaction, as shown in Fig. 4, is still phase matched. However, the sole contribution of the source term in Eq. (2a), in addition to the absence of any additional feedback mechanism, severely limits power conversion, that for ~600 nonlinear layers, remains below 0.01%. For the same number of layers in the photonic crystal case, the power conversion is above 7%. One may check the robustness of the conversion efficiency by introducing a 1% dispersion on the thickness of the linear and nonlinear layers, while the pitch of the periodicity is maintained. In such event, one observes no significant reduction in the efficiency of the process.

Conversion can be made very efficient if an alternative material with better nonlinear properties such as, for instance, Lithium niobate (LN) is considered. In such case, one may achieve 5% efficiency with the use of a very short crystal made of 100 nonlinear layers of ~100 nm long and a total crystal length of less than 20 μm. In that case, however, the high index contrast makes a clear separation of the counter from the copropagating contributions more difficult.

In conclusion, I have demonstrated that a broadband perfectly phase matched and efficient interaction of counterpropagating waves is possible in extremely short nonlinear 1D PC. With the current technology, periods of the order of less than a quarter of a visible wavelength should be achievable. In that event, one would be able to observe the reflectorless backward parametric oscillation predicted by Harris 39 years ago.[11] Such oscillators based on a counterpropagating interaction are likely to be more stable and perform more efficiently than the currently built oscillators based on a forward interaction where the mechanism of feedback is provided by a mirror cavity.

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